Double-Angle Formulas

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

Replace B by A,

$\sin(A+A) = \sin A \cos A + \cos A \sin A$

$\therefore \boxed{\sin 2A = 2 \sin A \cos A}$

$\cos 2A = \boxed{\cos^2 A - \sin^2 A}$

$= \cos^2 A - (1 - \cos^2 A) = \boxed{2\cos^2 A - 1}$

$= 1 - \sin^2 A - \sin^2 A = \boxed{1 - 2\sin^2 A}$

$\boxed{\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}}$

Find an expression for $\sin 3x$ in terms of $\sin x$.

$$\sin 3x = \sin(2x + x)$$
$$= \sin 2x \cos x + \cos 2x \sin x$$
$$= 2\sin x \cos x \cdot \cos x + (1 - 2\sin^2 x)\sin x$$
$$= 2\sin x \cos^2 x + \sin x - 2\sin^3 x$$
$$= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$$
$$= 3\sin x - 4\sin^3 x$$

Verify $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$

$$\frac{\sin 2x}{1 + \cos 2x} = \frac{2\sin x \cos x}{\cancel{1} + 2\cos^2 x \cancel{-1}}$$
$$= \frac{\cancel{2}\sin x \cancel{\cos x}}{\cancel{2}\cos x \cdot \cancel{\cos x}}$$
$$= \frac{\sin x}{\cos x} = \tan x$$

Graph $y = 2 - 4\cos^2 x$

Recall

$\cos 2x = 2\cos^2 x - 1$

$y = 2(\underbrace{1 - 2\cos^2 x})$

$y = -2(\underbrace{2\cos^2 x - 1})$

$y = -2\cos 2x$

S.P. $2x = 0$ $x = 0$ $y = -2$

MP $2x = \pi$ $x = \frac{\pi}{2}$ $y = 2$

E.P. $2x = 2\pi$ $x = \pi$ $y = -2$